Chapter 6
Analytical Aerotriangulation

6-1. General

Since ground control is a significant expense in any mapping project, aerotriangulation bridging or control extension methods are often used to reduce the amount of field surveying required by extending control to each stereomodel photogrammetrically. The number of control points required to scale and level each stereomodel does not change; on small projects of a few stereomodels, it may be cost-effective to establish all the stereomodel control by field survey methods. However, as the areal extent of a project increases, and thereby the number of stereomodels, aerotriangulation becomes an efficient method of extending a sparse field survey control network. This chapter emphasizes fully analytical aerotriangulation methods since these methods are most appropriate for modern instruments and large-scale mapping requirements.

6-2. Aerotriangulation Principles

a. Definition. Aerotriangulation is the simultaneous space resection and space intersection of image rays recorded by an aerial mapping camera. Conjugate image rays projected from two or more overlapping photographs intersect at the common ground points to define the three-dimensional space coordinates of each point. The entire assembly of image rays is fit to known ground control points in an adjustment process. Thus, when the adjustment is complete, ground coordinates of unknown ground points are determined by the intersection of adjusted image rays. An overview of aerotriangulation principles and methods is included in Chapter 10.

b. Purpose. The purpose of aerotriangulation is to extend horizontal and vertical control from relatively few ground survey control points to each unknown ground point included in the solution. The supplemental control points are called pass points, and they are used to control subsequent photogrammetric mapping. Each stereomodel is scaled and leveled using the adjusted coordinate values of the pass points located in the stereomodel.

c. Relationship to ground control. Aerotriangulation is essentially an interpolation tool, capable of extending control to areas between ground survey control using several contiguous uncontrolled stereomodels. An aerotriangulation solution should never be extended or cantilevered beyond the ground control. Ground control should be located at the ends of single strips and along the perimeter of block configurations. Within a strip or block, ground control is added at intervals of several stereomodels to limit error propagation in the adjusted pass point coordinates. Extending control by aerotriangulation methods is often referred to as *bridging* since the spatial image ray triangulation spans the gap between ground control.

6-3. Pass Points

Pass point requirements are related to type of point used, location, and point transfer and marking requirements.

a. Type of points. Pass points may be artificially marked points, targeted points, or natural images. However, since pass points must lie at or very near the center line of the triple overlap area, artificially marked points designed from the photography taken should be used. Premarked targets are too expensive and too difficult to align with the triple overlap areas. Natural images are not suitable for precise pointing.

b. Marking artificial points. Artificially marked pass points must be well-defined symmetrical patterns drilled, punched, or otherwise marked in the emulsion using a suitable marking instrument such as a Wild PUG or equivalent. Only the aerotriangulation/compilation positives should be marked. Normally, the original negatives should not be marked.

c. Location. A minimum of three pass points must be marked along the center line of each triple overlap area. One pass point must lie near the photo principal point, and one point must lie near each of the corners of the neat model. Pass point locations must be selected by examining the photographic prints with a stereoscope. Pass points must be located on unobscured level ground in accordance with the characteristics for vertical photo control. All pass point locations must be symbolized and labeled on the control photographs.

d. Point transfer for monoscopic measurements. Artificial pass points are typically marked in stereoscopic correspondence on all photographs showing the site of the point, using a stereoscopic transfer and point marking device such as a Wild PUG or equivalent. For the minimum three pass points in each triple overlap area, this operation will result in a minimum nine pass points on each photo. This method is required when photo coordinates are to be measured on a monocomparator. This method may be used when pass points are to be measured stereoscopically, either as photo coordinates on a stereocomparator or analytical stereoplotter or as model coordinates on any stereoplotter. Stereoscopic point transfer and marking should be done by a highly experienced operator using utmost care in choosing the site and in the marking of each pass point.

e. Point transfer for stereoscopic measurements. When pass points are to be measured stereoscopically, either as photo coordinates on a stereocomparator or analytical stereoplotter or as model coordinates on any stereoplotter, artificial pass points need be marked on the center photograph only using a suitable point marking device. Viewing the marked point stereoscopically with adjacent photographs will accomplish the point transfer of the pass point location to the overlapping photo as part of the measurement process. When parallel flight lines of photography are used, tie points should be transferred from one flight line to each adjacent flight line using a stereoscopic transfer and point marking device such as a Wild PUG or equivalent. Artificial points are typically not superimposed on images of targets.

6-4. Ground Control Points

Ground control requirements are related to targeting, control location, and survey accuracy requirements.

a. Targeting. Aerotriangulation for Class 1 and Class 2 mapping should require ground photo control points to be targeted prior to the flight. Targeting should be in accordance with paragraph 5-4a. Aerotriangulation for Class 3 mapping may use post-flight natural photo control points if approved by the Government.

b. Control location. Final control location and bridging distances used should be the decision of the contractor, but the following guidelines should be applied:

(1) Single strips

(a) Along a single flight line of photography (Figure 6-1), horizontal control points should be in pairs at the strip ends within the terminal stereomodel, one on each side of the flight line approximately opposite each other. For each single flight line, additional horizontal control points should be located at intervals along the strip that do not exceed the maximum allowable bridging distance. Horizontal basic control points should be no more than one-third of the width of coverage of a photograph from the flight line.

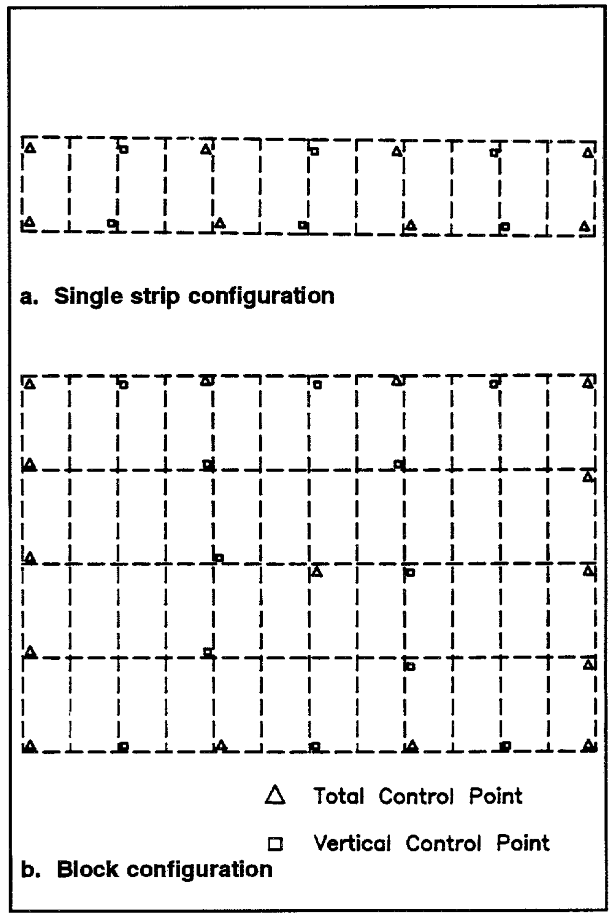

Figure 6-1. Typical strip and block control configurations

(b) Along a single flight line of photography, vertical control points should be in pairs, one on each side of the flight line approximately opposite each other and at a distance from the flight line of between one-fourth and one-third of the width of coverage of a photograph. For each single flight line, the pairs of vertical control points should be located at the strip ends within the terminal stereomodel and at intervals along the strip that do not exceed the allowable bridging distance for the class of mapping.

(2) Blocks. A block of photography, consisting of two or more flight lines of photography (Figure 6-1), should have control points spaced approximately equally

around the periphery following the same spacing and location guidelines as for strips in paragraph 6-4.b(1). There should be at least one horizontal control point and two vertical control points near the center of any block.

(a) Additional horizontal control should be located in the center of the block such that horizontal control falls in alternating strips at an interval not to exceed two times the allowable horizontal bridging distance. There should be at least one horizontal control point and two vertical control points near the center of any block.

(b) Additional vertical control should be located in the center of the block such that vertical control falls in each strip at an interval not to exceed two times the allowable vertical bridging distance.

c. Bridging distance. Table 6-1 lists typical allowable bridging distances that may be used as a guide in estimating control requirements for a project. These are minimum guidelines, and many contractors will design a more dense control pattern. For example, a horizontal control point every five stereomodels regardless of block size.

Table 6-1
Allowable Bridging Distances

	Allowable Bridging Distance (Stereomodel Bases)	
Map Class	Maximum Horizontal Control Spacing	Maximum Vertical Control Spacing
1	4	2
2	5	3
3	6	3

d. Ground control accuracy. Ground control accuracy for aerotriangulation should be more stringent than for a project fully controlled by field survey points.

The aerotriangulation solution will contribute to the propagated error in the pass point ground control values. Since the pass point coordinates should meet the accuracy required for photo control, the photoidentifiable ground control points used to control the aerotriangulation should be more typical of the accuracy of the basic control survey.

e. GPS control. The GPS is an effective method of establishing basic project control and photo control. GPS is an especially effective way to connect the project area surveys to existing national network stations outside the project region. Kinematic GPS methods are also being used to position the camera at the time of exposure. If the camera position is known, ground control points used to resect for the camera position can be eliminated. The use of kinematic GPS methods should be evaluated carefully. Unless the terrain is inaccessible for ground targeting, terrestrial GPS surveys to establish control points and normal flying procedures may be more cost-effective and accurate for large-scale mapping. GPS theory and applications are more fully discussed in Leick (1990).

6-5. Other Points

Coordinates can be established by aerotriangulation for additional points located on the photography by targets or artificially marked as pass points. For example, these points may be aerotriangulation checkpoints, stereoplotter test points, or cadastral points to be located on the map. The Government should specify all points required to be included in the aerotriangulation in addition to the control points and standard pass point pattern. If a supplemental pass point is required for checking stereomodel flatness at compilation time, it should be located near the center of the stereomodel within a circle whose diameter is the central third of the air base.

6-6. Instrumentation

Precise photo coordinate measurements are required for fully analytical aerotriangulation. An analytical stereoplotter or a monocomparator must be used. When semianalytical aerotriangulation serves as a preadjustment for fully analytical aerotriangulation, instrumentation and measurement procedures are controlled by the requirements for fully analytical aerotriangulation. When semianalytical aerotriangulation is to be the final adjustment for pass point coordinates, coordinates of independent stereomodels must be measured on a Second-Order analog instrument or better as defined in Table 7-1.

6-7. Accuracy and Quality Control Criteria

The contractor is responsible for designing the aerotriangulation scheme that will meet the requirements of the photogrammetric product. Table 6-2 summarizes the guidelines for evaluating aerotriangulation methods. However, since meeting required pass point accuracies is dependent on the photogrammetric system, the contractor should be allowed some latitude in meeting criteria for these intermediate results.

Table 6-2
Guidelines for Evaluating Analytical Aerotriangulation

Analytical Aerotriangulation Procedures	Criteria
Photo Coordinate Measurements: Monocomparator or Analytical Stereoplotter	
Least Count of Stage Coordinate	0.001 mm
Interior Orientation: Transformation to Fiducial Coordinates	
Minimum Number of Fiducials	
Maximum Residual (After Affine Transformation)	4
	0.020 mm
Preliminary Sequential Strip Formation and Adjustment	
Stereomodel Relative Orientation	
Minimum Number of Points, y-parallax Residuals	6
RMSE	0.005 mm
Maximum	0.015 mm
Stereomodel Joins	
Minimum Number of Points	3
x,y Pass Point Coordinate Discrepancy	
RMSE	H/12,000 ft
Maximum	H/ 6,000 ft
z Pass Point Coordinate Discrepancy	
RMSE	H/10,000 ft
Maximum	H/ 5,000 ft
Polynomial Strip Adjustment	
X,Y Control Point Coordinate Residual	
RMSE	H/10,000 ft
Maximum	H/ 6,000 ft
Z Control Point Coordinate Residual	
RMSE	H/ 7,000 ft
Maximum	H/ 5,000 ft
Simultaneous Bundle Adjustment	0.004 mm
RMSE of Photo Coordinate Residual	1.5
Maximum Variance Factor Ratio	
(See also Table 6-3)	

a. Photo coordinate measurements. Photo coordinate measurement is the most critical factor contributing to the accuracy of aerotriangulation results. The contractor should be especially careful to control the quality of point transfer, point marking, and point measurement. The measurement stage(s) of the stereoplotter or the monocomparator should have a least count of 0.001 mm or less. The viewing, pointing, and digitizing components of these instruments should enable the operator to group multiple readings on any well-defined target or marked pass point within a maximum spread of 0.004 mm. Multiple readings are of more value if they are not consecutive; however, this reading scheme is often not practical. If multiple readings are made consecutively, the operator must move off the image and re-point between each reading. The instrument used should be capable of measuring a photo coordinate with an RMSE not greater than 0.003 mm. It is mandatory to end the measuring of a photograph with a reading on the first point measured (usually a fiducial mark) to assure that the instrument encoders have not drifted or skipped counts.

b. Interior orientation. Interior orientation refers to the geometric relationship between the image plane and the perspective center of the lens.

(1) The initial step is to transform the measured stage coordinates into the photo coordinate system defined by the calibrated fiducial coordinates. An affine transformation, which accounts for differential image scale and shear, is typically used to establish the photo

coordinate system. The transformation parameters are determined by a least squares adjustment using at least the four midside or four corner fiducials. If both types of fiducials are present in the camera, eight fiducial points can be used to increase the redundancy of the adjustment.

(2) After the photo coordinate system is established, the image measurements must be corrected for systematic errors. This procedure is called photo coordinate refinement. Corrections are applied for principal point offset, radial lens distortion, tangential lens distortion, and atmospheric refraction. Procedures for making these corrections may be found in Wolf (1983) and Moffitt and Mikhail (1980). Correction for tangential lens distortion may be neglected if it is found to be insignificant in the camera calibration report. Photo coordinate refinement may be performed by the analytical stereoplotter software or the aerotriangulation software. Typically, the interior orientation and refinement parameters are considered known based on the calibration report. Then the photo coordinate refinement is performed before the photo coordinates are used for the aerotriangulation adjustment. If self-calibration aerotriangulation software is used, the camera interior orientation parameters are considered to be approximations, and they are adjusted as a parameter in the aerotriangulation solution.

c. Preliminary sequential aerotriangulation. This process (Table 6-2) refers to the sequential assembly of independent stereomodels to form a strip unit and the polynomial strip adjustment into the ground coordinate system. The sequential procedure is a preliminary adjustment that develops initial approximations for the final simultaneous bundle adjustment. The sequential procedure also serves as a quality control check of the photo and ground coordinate data. The guidelines listed in Table 6-2 are not rigorously enforced, but they are used to evaluate the building blocks of a larger strip or block configuration.

(1) Relative orientation of each stereo pair is performed by a least squares adjustment using the collinearity equations. The stereomodel is created in an arbitrary coordinate system, and the adjustment is unconstrained by ground coordinate values. Therefore, the photo coordinate residuals should be representative of the point transfer and measuring precision. The photo coordinate residuals should be examined to detect misidentified or poorly measured points. The minimum number of points that will uniquely determine a relative orientation is five, and the photo coordinate residuals will be zero. The six-point minimum recommended in Table 6-2 results if the standard nine pass points per photo configuration is used. With only one redundant pass point, the photo coordinate residuals will still be quite small unless a large blunder is present. Typically, more than the minimum six pass points are used. The RMSE and maximum residual values listed will more likely be reached when larger numbers of pass points are used.

(2) When stereomodels are joined to form a strip, the pass points shared between models will have two coordinate values, one value in the strip coordinate system and one value in the transformed model coordinate system. The coordinate differences or discrepancies between the two values can be examined to evaluate how well the models fit to one another. Horizontal coordinate discrepancies will typically be smaller than vertical discrepancies since the image ray intersection geometry is weaker in the vertical direction. As the stereomodels are transformed into the strip, one after the other, the pass point coordinate discrepancies should be uniform and no outliers should be observed. The coordinate discrepancy criterion is expressed as a fraction of the flying height above terrain because the magnitude of the discrepancy in ground units is dependent on the photo scale.

(3) Polynomial strip adjustment is a preliminary adjustment that produces initial ground coordinate values for all the pass points in a strip. The pass point coordinate values will be adjusted again by the final bundle adjustment. The polynomial correction curve is fit to the coordinate errors at the control point locations using a least squares adjustment. The residuals of the least squares curve fit can be examined to evaluate the adequacy of the polynomial adjustment. The residual criterion is expressed as a fraction of the flying height above terrain because the magnitude of the discrepancy in ground units is dependent on the photo scale. The evaluation of the polynomial residuals is the least critical check in the aerotriangulation process, and a great deal of latitude can be allowed in meeting these criteria. From project to project, large variations in the residuals may occur because of the number of stereomodels in the strip, the polynomial function used, and the distribution of the control points. It is more important to check the X, Y, and Z error curves after the linear transformation of the strip into the ground coordinate system and before the polynomial correction. These error curves should be smooth Second- or Third-Order curves. Outliers from a smooth continuous curve are an indication

of a blunder in the photogrammetric value or the ground survey value at a control point.

d. Simultaneous bundle adjustment. Fully analytical aerotriangulation must be adjusted by a weighted least squares adjustment method. The adjustment software must form the collinearity condition equations for all the photo coordinate observations in the block and solve for all photo orientation and ground point coordinates in each iteration until the solution converges.

(1) The exterior coordinate system used for the adjustment should be a local rectangular coordinate system as defined in Chapter 10. This coordinate system contains no earth curvature or map projection distortions. These effects may be judged to be negligible for small project areas and low flying heights, but they are significant factors for large project areas and high flying heights.

(2) The least squares adjustment results should be examined to check the consistency of the photo coordinate measurements and the ground control fit. Residuals on the photo coordinates should be examined to see that they are representative of the random error expected from the instrument used to measure them. Residuals should be randomly plus or minus and have a uniform magnitude. The residuals should be checked carefully for outliers and systematic trends. The standard deviation of unit weight computed from the weighted adjusted residuals should not be more than 1.5 times the reference standard deviation used to compute the weights for the adjustment. A large computed reference variance indicates inflated residuals and possible systematic errors affecting the adjustment. For example, if photo coordinates were judged to have an overall measurement standard deviation of 0.005 mm and this value was used to compute observation weights, the standard deviation of unit weight computed by the adjustment should not exceed 0.0075 mm. A complete discussion of least squares adjustment methods and analysis of adjustment results can be found in Mikhail (1976).

(3) Accuracy of aerial analytical triangulation should be measured by the RMSE and the maximum error in each coordinate (X, Y, and Z) direction for the combined checkpoints. The maximum allowable error should be checked at the midpoint of the bridging distance between ground control points using checkpoints or test drop points surveyed for this purpose. Table 6-3 lists the accuracy criteria suggested for each class of mapping. These criteria are the final and most important check of the aerotriangulation results.

e. Semianalytical aerotriangulation. When semianalytical aerotriangulation is to be the final adjustment for pass point coordinates, final adjustment of strips or blocks must be performed using numerical polynomial strip adjustment or simultaneous stereomodel block adjustment. Analog or graphical adjustment methods must not be used. This criterion includes analytically computed independent stereomodels using monocomparator photo coordinates. Strips must be assembled by numerical coordinate transformations of successive stereomodels. Strips of stereomodels may be assembled using base-in/base-out capabilities on analytical or universal analog stereoplotters.

Table 6-3
Aerotriangulation Accuracy Criteria (for 6-in. Focal Length Photography)

		Allowable RMSE at Test Points[1]	
Map Class	Aerotriangulation Method	Horizontal[2]	Vertical[2]
1	Fully Analytical	H/10,000	H/9,000
2	Fully Analytical	H/8,000	H/6,000
3	Fully Analytical or Semianalytical	H/6,000	H/4,500

Notes:
[1] The maximum allowable error is 3 RMSE.
[2] One-sigma level.

6-8. Stereoplotter Settings

Fully analytical aerotriangulation determines the six camera exterior orientation parameters for each photograph, camera position (X_L, Y_L, and Z_L), and angular orientation (the omega-phi-kappa system described in paragraph 10-6). By relating these parameters to the flight line between each two successive camera stations and scaling to the stereomodel, data are obtained for setting up the stereoscopic model in the stereoplotter. For a specific analog stereoplotter, orientation settings can be derived that will enable the operator to set up the stereomodel for compilation much faster than the manual empirical method, thereby saving expensive stereoplotter time during the compilation phase of the project.

6-9. Deliverables

Unless otherwise modified by the contract specifications, the following materials will be delivered to the Government upon completion of the aerotriangulation:

a. General report about the project and procedures used including description of the project area, location, and extent; description of the instrumentation used for pass point transfer and marking, and photo coordinate measurement; and description of the aerotriangulation methods and software used including version numbers.

b. One set of paper prints showing all control points and pass points used. The points should be symbolized and named on the image side, and the exact point location should be pinpricked through the print.

c. A list of the computed coordinates of all points specified by the Government.

d. A report of the accuracies attained and listing discrepancies in each coordinate direction at control points and checkpoints separately, a justification for any control points or pass points omitted from the final adjustment, and the RMSE and maximum error (in relation to ground surveyed coordinates) in each coordinate direction (X, Y, and Z) for the control points and checkpoints as a group.

e. Complete copies of all computer printouts (Figure 6-2).

f. A list of stereoplotter orientation settings, if specified.

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POINT NO.	EASTING	NORTHING	ELEVATION	D-EAST	D-NORTH	D-ELEV	
556	2786827.385	250839.112	2412.14	-0.165	-0.028	0.28	H & V
3521	2788659.065	251320.650	2411.14				
3522	2787483.426	251227.729	2408.32				
3523	2786402.842	250902.489	2414.15				
4621	2787457.352	232749.686	2392.55				
4622	2786331.083	232803.281	2401.21				
4623	2785144.880	232809.955	2422.71				
580	2786204.833	233229.089	2405.01	0.053	0.169	0.09	H & V
558	2785370.182	233994.918	2422.22	0.082	-0.092	0.09	H & V
4631	2787471.758	234229.074	2400.87				
4632	2786396.453	234204.204	2406.73				
4633	2785372.762	234188.906	2424.67				
4641	2787462.665	235694.789	2406.46				
4642	2786362.539	235646.811	2418.74				
4643	2785324.900	235526.918	2429.87				
559	2785316.662	235848.956	2429.34	0.222	0.026	0.06	H & V

ROOT MEAN SQUARE ERRORS

	HORIZONTAL				VERTICAL	
	NO. POINTS	X	Y	RESULTANT	NO. POINTS	Z
HELD	27	0.14	0.17	0.22	62	0.17
KNOWN, BUT NOT HELD	3	1.54	0.82	1.75	0	0.0
ALL KNOWN POINTS	30	0.50	0.30	0.59	62	0.17

Figure 6-2. Portion of output from analytical aerotriangulation adjustment (from US Army Engineer District, Seattle)